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Sticky Prices or Sticky Wages? An Equivalence Result*

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Abstract

We show an equivalence result in the standard representative agent New Keynesian model after demand shocks: assuming sticky prices and flexible wages yields identical allocations for GDP, consumption, labor, inflation and interest rates to the opposite case—flexible prices and sticky wages. This equivalence result arises if the price and wage Phillips curves' slopes are identical and generalizes to any pair of price and wage Phillips curve slopes such that their sum and product are identical. Nevertheless, the cyclical implications for profits and wages are substantially different. We discuss how the equivalence breaks when these factor-distributional implications matter for aggregate allocations, e.g. in New Keynesian models with heterogeneous agents, endogenous firm entry, and non-constant returns to scale in production.

JEL Classification: E1, E3, E5.

Keywords: Sticky prices, sticky wages, New Keynesian model, inflation, output, interest rate, observational equivalence.

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1 Introduction

In this note, we point out an equivalence result in the standard representative agent New Keynesian model that is to the best of our knowledge novel to the literature. We show in closed form that assuming sticky prices and flexible wages yields observationally equivalent allocations for GDP, consumption, labor, inflation and interest rates to the case when assuming flexible prices and sticky wages instead. The observational equivalence result arises after demand shocks, e.g. monetary policy, government spending and discount factor shocks, if the slope of the respective price and wage Phillips curves are identical. The generalized version of our key result is that any arbitrary combination of price and wage Phillips curve slopes yields the same equivalence, as long as their sum and product are identical.

Despite the equivalent aggregate allocations, we show that the two polar cases of sticky prices/flexible wages and flexible prices/sticky wages have markedly different implications for the cyclicality of profits and real wages: in other words, although aggregate income is identical, the distribution of income across factors is very different over the business cycle. We discuss these implications and end by pointing out causes of breakup of the equivalence. The source of such breakup is a feedback—common to several existing contributions in the literature—from the distribution of income across factors to aggregate income: e.g. under household heterogeneity, under firm entry, or under non-constant returns to scale in production.

The note is organized as follows. Section two lays out the model and establishes our key observational equivalence result. Section three discusses sources of breakup of the observational equivalence result and the connection to the literature. Finally, section four concludes.

2 Model and Equivalence Result

We show our equivalence result in the standard textbook representative agent New Keynesian model with sticky prices and sticky wages, see e.g. Erceg, Henderson and Levin (2000), Woodford (2003), Galí (2015), and Walsh (2017).

2.1 Model

We work with the same log-linearized equilibrium equations used in the references above; in particular, the only difference relative to Galí (2015) is that we take the special case of constant returns to scale in production, no TFP ("supply") shocks and no endogenous capital accumulation—we

consider some of these extensions below. We consider the set of "demand" shocks to include monetary policy shocks, v_t , government spending, around a zero steady-state value (so g_t is defined in shares of steady-state output, $g_t \equiv (G_t - G)/Y$) and household discount factor shocks, δ_t . Finally, we let $\sigma \geq 0$ denote the elasticity of intertemporal substitution in consumption, i.e. the inverse of risk aversion. The log-linearized equilibrium equations of the model read as follows:

Price Phillips Curve : $\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p m c_t$

Wage Phillips Curve : $\pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w \left[\omega_t - (\sigma^{-1} c_t + \varphi n_t) \right]$

Real wage growth definition : $\omega_t = \omega_{t-1} + \pi_t^w - \pi_t^p$

New-IS Curve : $c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}^p) + \delta_t$

Taylor rule : $i_t = \phi_\pi \pi_t^p + v_t$

Production and ERC : $c_t + g_t = n_t = y_t$

Marginal cost : $mc_t = \omega_t$

The first equation is the New Keynesian Price Phillips curve relating the price inflation rate π_t^p to its expected value and to real marginal cost mc_t , with slope $\lambda_p \geq 0$. The second equation the New Keynesian Wage Phillips curve relating the wage inflation rate π_t^w to its expected value and the difference (sometimes called "wage markup") between the real wage ω_t and the marginal rate of substitution between consumption c_t and hours worked n_t ; $\lambda_w \geq 0$ is the slope of the Wage Phillips curve, and $\varphi \geq 0$ the inverse Frisch labor supply elasticity. The third equation derives from the definition of the real wage: its growth rate is the difference between (nominal) wage and price inflation. The fourth equation is the New-IS curve relating expected consumption growth to the ex-ante real interest rate, defined as the nominal interest rate i_t net of expected inflation. The fifth equation is a Taylor-type feedback rule relating nominal interest to realized inflation with reaction coefficient $\phi_{\pi} \geq 1$; v_t is an exogenous monetary policy shock or, more generally, a demand shock. The sixth equation combines the linear production function for output y_t and the economy resource constraint (ERC, with zero steady-state public spending). The last equation is the marginal cost of production, which is merely equal to the real wage under constant returns and no TFP shocks.

2.2 Equivalence Result

In this section, we derive analytically derive the observational equivalence result. In a first step, we consider two polar cases: i) sticky prices/flexible wages and ii) flexible prices/sticky wages. In a second step we generalize our key observational equivalence result to arbitrary combinations of

price and wage stickiness.

With sticky prices/flexible wages, by virtue of the latter feature, the slope of the Wage Phillips curve goes to infinity $\lambda_w \to \infty$ and the Wage Phillips curve equation becomes:

$$\omega_t = \sigma^{-1}c_t + \varphi n_t.$$

Using this equation together with the remaining (unchanged) equilibrium equations yields, after simplification:

Price Phillips Curve :
$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p (\sigma^{-1} + \varphi) c_t + \lambda_p \varphi g_t$$
 (1)

New-IS Curve :
$$c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}^p) + \delta_t$$
 (2)

Taylor rule :
$$i_t = \phi_\pi \pi_t^p + v_t$$
 (3)

With **flexible prices/sticky wages**, by virtue of the former feature, the slope of the Price Phillips curve becomes infinite $\lambda_p \to \infty$ and the Price Phillips curve becomes:

$$mc_t = \omega_t = 0$$

Using this equation together with the remaining (unchanged) equilibrium equations yields, after simplification:

Wage Phillips Curve :
$$\pi_t^p = \beta E_t \pi_t^p + \lambda_w \left(\sigma^{-1} + \varphi\right) c_t + \lambda_w \varphi g_t$$
 (4)

New-IS Curve :
$$c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}^p) + \delta_t$$
 (5)

Taylor rule :
$$i_t = \phi_\pi \pi_t^p + v_t$$
 (6)

We are now ready to establish our first key result, emphasized in the following Proposition.

Proposition 1: Equivalence. If the slopes of the Price and Wage Phillips curves are the same, $\lambda_w = \lambda_p$, the aggregate allocations for π_t^p , c_t , i_t and n_t (and y_t) are identical—in response to monetary shocks v_t , government spending shocks g_t , or discount factor shocks, δ_t —regardless of whether one assumes sticky prices/flexible wages or flexible prices/sticky wages.

The proof follows directly by observing that equations (1)-(3) and (4)-(6) are identical if $\lambda_w = \lambda_p$.

Proposition 1 is in fact a special case of the following more general equivalence Proposition.¹

¹We are grateful to Jordi Galí and Tobias Broer for encouraging us to consider the general equivalence case.

Proposition 2: General equivalence. The aggregate allocations for π_t^p , c_t , i_t and n_t (and y_t) are identical—in response to monetary shocks v_t , government spending shocks g_t , or discount factor shocks, δ_t — if for any pairs of slope parameters $\{\lambda_p, \lambda_w\}$ and $\{\lambda_p', \lambda_w'\}$ of the price and wage Phillips curves the following two conditions are satisfied:

$$\lambda_{w} + \lambda_{p} = \lambda'_{w} + \lambda'_{p}$$

$$\lambda_{w} \lambda_{p} = \lambda'_{w} \lambda'_{p},$$

$$(7)$$

i.e. for any pair of slopes $\{\lambda_p, \lambda_w\}$ such that their sum $\lambda_p + \lambda_w$ and product $\lambda_p \lambda_w$ are constant. In other words, equivalence obtains either if (trivially) the slopes are pairwise identical, or if they are flipped:

$$\lambda_{w}^{'} = \lambda_{p} \text{ and } \lambda_{p}^{'} = \lambda_{w}.$$
 (8)

The proof is as follows. In the Appendix we show that the original system of equations in section (2.1) can be rewritten as follows:

Phillips Curves:
$$[1 + \lambda_p + \lambda_w] \tilde{\Delta} \pi_t^p = \beta \left(E_t \tilde{\Delta} \pi_{t+1}^p - \tilde{\Delta} \pi_t^p \right) + \tilde{\Delta} \pi_{t-1}^p + [\lambda_w \lambda_p] \left(\left(\sigma^{-1} + \varphi \right) c_t + \varphi g_t \right)$$

New-IS Curve : $c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}^p) + \delta_t$

Taylor rule : $i_t = \phi_\pi \pi_t^p + v_t$

where $\tilde{\Delta}\pi_t^p \equiv \pi_t^p - \beta E_t \pi_{t+1}^p$. The first equation combines the price and wage Phillips curves, the law of motion for the real wage, the resources constraint and the definition of marginal cost. Note that the resulting set of equilibrium equations displayed above yields observationally equivalent allocations for π_t^p , c_t , i_t (and also n_t and y_t) to the three shocks in the model for any combination of λ_w and λ_p iff the numerical values implied by the two square brackets in the 'Phillips Curves' equation remain unchanged. In other words, $\lambda_p + \lambda_w$ and $\lambda_w \lambda_p$ must remain unchanged when varying λ_w and λ_p which provides a proof of (7) in Proposition 2. The final part of proposition 2 in (8) follows from solving the system of equations implied by (7) $\lambda_w + \lambda_p = a$ and $\lambda_w \lambda_p = b$; the two solutions are either (trivially) constant λ_p and λ_w , or flipped $\lambda_p = \lambda_w$ and $\lambda_w = \lambda_p$.²

²One implication of our equivalence result is that there is an identification problem for wage and price stickiness: based on aggregate data on inflation, consumption, hours, output, and interest rates alone, it is impossible to disentangle between these two parameters; one needs to inform the estimation by data on wage inflation, real wages, and/or profits. Interestingly, many estimated New Keynesian models include such data, see e.g. Christiano et al. (2005).

Note, however, that even in the observational-equivalence case (be it our two polar cases or the general case) real wages (and thus wage inflation) and profits are different between the two equilibria. So even if equilibrium production and income are identical, the distribution of income between production factors is different—which further implies that the equivalence naturally breaks whenever the distribution of income between factors has an allocative role.

3 Differences and Sources of Equivalence Breakup

In this section, we first discuss the different implications for the cyclicality of real wages and profits, even in the case where observational equivalence holds; we then briefly outline the sources of equivalence breakup and connection to the literature.

3.1 Cyclicality of Wages, Profits, and Wage Inflation

Note that while allocations for π_t^p , c_t , i_t and n_t (and y_t) are identical regardless of which of the two polar cases for nominal rigidities is assumed, the dynamics of wages, profits, and wage inflation differ across the two cases. We focus on the two polar cases for clarity of exposition compared to the general case.

Denote the share of profits in steady-state output by:

$$\frac{D}{PY} = 1 - \frac{WL}{PY} = 1 - \frac{1}{\mathcal{M}} \ge 0,$$

where \mathcal{M} is the post-subsidy gross markup (which tends to unity under an optimal sales subsidy). Letting profits d_t be expressed as deviations in share of steady-state output (to accommodate possible zero-steady-state value, just as for g_t), the linearized definition of profits is:

$$d_t = y_t - \frac{1}{\mathcal{M}} \left(\omega_t + n_t \right)$$

Consider again the two polar cases. With **sticky prices/flexible wages**, real wages and equilibrium profits read:

$$\omega_t = (\sigma^{-1} + \varphi) y_t - \sigma^{-1} g_t$$

$$d_t = \left(1 - \frac{1 + \varphi + \sigma^{-1}}{\mathcal{M}}\right) y_t + \frac{\sigma^{-1}}{\mathcal{M}} g_t$$

It follows that real wages are procyclical and profits are countercyclical to demand shocks, generically (insofar as $\mathcal{M} - 1 < \varphi + \sigma^{-1}$, which is plausible in the data). Moreover, wage inflation is

$$\pi_t^w = \pi_t^p + \omega_t - \omega_{t-1} = \pi_t^p + \left(\sigma^{-1} + \varphi\right) \Delta y_t - \sigma^{-1} \Delta g_t$$

so in response to expansionary monetary shocks, wage inflation is higher than price inflation.³

Now, consider the case of flexible prices/sticky wages. In this case ($mc_t = 0$), equilibrium real wages and profits read:

$$\omega_t = 0
d_t = \left(1 - \frac{1}{\mathcal{M}}\right) y_t,$$

so profits are procyclical, wages acyclical, and wage inflation is equal to price inflation.

In other words, while the two polar cases for nominal rigidities imply identical allocations for π_t^p , c_t , i_t and n_t (and y_t), they have markedly different implications for the distribution of income between factors. Note that empirically, profits are procyclical after demand shocks, see e.g. Christiano, Eichenbaum and Evans (1997, 2005); the point estimate of the response of the real wage to a demand shock is slightly procyclical although the confidence bands are so wide, that the real wage response is typically considered a-cyclical after a demand shock. To render a standard New Keynesian model compatible with procyclical profits after demand shocks, it is enough to have wages sufficiently sticky relative to prices, see e.g. Christiano, Eichenbaum and Evans (2005).^{4,5}

An important nuance regarding the monetary transmission mechanism arises as a natural implication of our equivalence result. Broer et al. (2020) argue that due to the countercyclicality of profits, the standard textbook New Keynesian model with sticky prices and flexible wages implies or indeed relies upon an "implausible" monetary transmission mechanism. Our equivalence result provides a counterpoint to this interpretation, at least when it comes to the aggregate allocation. Specifically, even though profits are countercyclical or procyclical to demand shocks, the aggregate allocations for π_t^p , c_t , i_t , n_t and y_t are identical—in other words, whether profits are pro- or countercyclical is irrelevant. This admittedly extreme illustration challenges the notion that the

³A direct corollary of this is that if the central bank responded (in the Taylor rule) to wage inflation rather than price inflation, equivalence would break down; we thank Erik Öberg for pointing this out. Relatedly, under optimal monetary policy the equivalence breaks down too, since under wage stickiness there is an additional wage-inflation stabilization motive (see Erceg et al. 2000, Gali 2015). Erceg et al. (1998) study optimal policy comparing explicitly the same two polar cases we consider.

⁴For a recent detailed analysis of the cyclicality of profits, both in the data and deriving an analytical condition on the degree of wage stickiness required to render profits procyclical under decreasing returns, see Bilbiie and Känzig (2023). See Cantore et al. (2020) for evidence and an analytical result on the cyclicality of real wages and the labor share.

⁵In a recent related contribution, Broer, Harmenberg, Krusell, and Öberg (2023) study the connection between models of rigid wage contracts and rigid wages, and the smilarities with a flexible-wage model with preferences featuring habits and intertemporal wealth effects.

cyclicality of profits is of first-order importance for aggregate allocations in the standard textbook New Keynesian model in and of itself. At the same time, this is of course of the essence in models where the distribution of profits matters for the allocation, which are models where our equivalence result breaks down (because of e.g. agent heterogeneity, firm entry, or non-constant returns to scale in production).

3.2 Breaking the Equivalence and Connection to Literature

We now discuss the model features that break the equivalence we derived and thus make the distinction between price and wage stickiness meaningful. The first subclass deals with features of the standard textbook New Keynesian model. Under decreasing returns to scale and TFP shocks (the benchmark model in the Galí and Woodford textbooks), we show in the Appendix that the isomorphism no longer applies, even though the "demand" sides (the Euler equation and Taylor rule) are completely identical. The reason is that the price and wage Phillips curves are different: the latter always contains a term that captures the difference between consumption and hours growth, and this term is always non-zero insofar as either returns to scale are non-constant or there are disturbances to TFP.⁶

The second subclass of equivalence breakups pertains to model extensions, in particular house-hold heterogeneity and endogenous entry. Generally, the equivalence breaks in a wide class of heterogeneous-agent New Keynesian models, be in their early incarnations with two agents or in their more recent quantitative versions modelling the full distribution. In all such models, the distribution of profits—and hence their cyclicality—not only matters but is indeed paramount for the propagation of demand shocks: see Bilbiie (2008) for an early version of that argument in a two-agent model. Colciago (2011), Ascari et al. (2017), Bilbiie et al. (2022), and Diz et al. (2023) study sticky-wage extensions of that two-agent model.

Broer et al. (2020) provide a sharp illustration of the complete lack of monetary transmission under flexible wages when workers receive none of the profits, and thus a justification for wage stickiness in generating a more plausible monetary transmission mechanism; Broer et al. (2023) extend that analysis to the implications for fiscal multipliers. Quantitative heterogeneous-agent models focusing on the role of wage stickiness include, e.g. focusing on fiscal multipliers Auclert et al. (2019), Auclert et al. (2023), and Hagedorn et al. (2019), and focusing on monetary policy Alves et al. (2021) and Hagedorn et al. (2019).

⁶A similar breakup of the equivalence occurs in the presence of endogenous investment in physical capital.

The general intuition of all such heterogeneous-agent models is that not all households are on their Euler equation and (thus) not all agents receive profit income from share holding: it thus becomes essential for the aggregate(d) demand side of the model what agents (low- or high-MPC) perceive what is income, and what are the cyclical properties of that income. More importantly, whether wages are flexible or sticky matters a great deal for aggregate dynamics. See e.g. Bilbiie and Känzig (2023) for recent work on wage versus price stickiness, profits cyclicality, and the connection with the dynamics of aggregate demand and inflation in heterogeneous-agent models.

The equivalence result also breaks down under endogenous entry, because now the dynamics and cyclicality of (per-firm) profits drive the entry and exit decision: countercyclical profits (sticky prices) thus imply exit, while procyclical profits (sticky wages) imply entry, with marked differences for the full model dynamics.⁷

4 Conclusion

We have shown analytically an equivalence result in the standard representative agent New Keynesian model: assuming sticky prices and flexible wages yields observationally equivalent allocations for GDP, consumption, labor, inflation and interest rates to the case when assuming flexible prices and sticky wages instead. The observational equivalence result arises after demand shocks (such as shocks to monetary policy, government spending, or the household discount factor) if the slopes of the respective price and wage Phillips curves are identical. The general version of this equivalence obtains for any combinations of price and wage Phillips curve slopes such that their sum and product are constant—or, in other words, merely inverting the values of the price and wage Phillips curve slopes leaves those aggregate variables unchanged.

We have discussed implications for the cyclicality of profits and real wages, monetary transmission, New Keynesian models with heterogenous agents, endogenous firm entry and non-constant returns to scale in production.

At a more general level, our equivalence result provides an additional element to the impetus for the shifting focus of the New Keynesian literature towards wage stickiness—from Christiano et

⁷One illustration of this breakup can be seen under free, frictionless entry with Dixit-Stiglitz preferences (see Bilbiie, 2021): under sticky prices and free entry there is in fact neutrality, as the extensive and intensive margins move in offsetting ways in response to demand shocks, because entry is efficient. But under sticky wages and free entry, a Phillips curve re-emerges that is isomorphic to the sticky-price no-entry one—which, as our note has just shown, is also isomorphic to the sticky-wage no-entry case under the assumptions we spelled out. See Bilbiie and Melitz (2020) for the full aggregate implications of wage stickiness with endogenous entry and exit.

al. (2005) and Galí (2011), to the plethora of (micro and time series) evidence regarding its realism; to the heterogeneous-agent, distributional-based motivation in Auclert et al. (2019), Broer et al. (2020), and Hagedorn et al. (2019), to the entry-exit implications in Bilbiie and Melitz (2020). All these contributions point towards relative benefits of wage—in addition to or even instead of—price stickiness for various business-cycle properties and comovements. Our note provides an analytical perspective on its tractability and closeness to the sticky-price-only benchmark.

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Appendix A Deriving the General Equivalence Case

Starting with the original equilibrium equations provided in section (2.1) above, substitute out marginal cost and the production/ERC to get:

Price Phillips Curve : $\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \omega_t$

Wage Phillips Curve : $\pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w \left(\omega_t - \left(\left(\sigma^{-1} + \varphi \right) c_t + \varphi g_t \right) \right)$

Real wage growth definition : $\omega_t = \omega_{t-1} + \pi_t^w - \pi_t^p$

New-IS Curve : $c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}^p) + \delta_t$

Taylor rule : $i_t = \phi_\pi \pi_t^p + v_t$

Next, solve the Price Phillips curve for ω_t and substitute into the Wage Phillips curve and the real wage growth definition:

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w \left(\frac{\pi_t^p - \beta E_t \pi_{t+1}^p}{\lambda_p} - ((\sigma^{-1} + \varphi) c_t + \varphi g_t) \right)$$

$$\frac{\pi_t^p - \beta E_t \pi_{t+1}^p}{\lambda_p} = \frac{\pi_{t-1}^p - \beta \pi_t^p}{\lambda_p} + \pi_t^w - \pi_t^p$$

Finally, substituting for π_t^w yields:

$$\frac{\pi_t^p - \beta E_t \pi_{t+1}^p}{\lambda_p} - \frac{\pi_{t-1}^p - \beta \pi_t^p}{\lambda_p} + \pi_t^p = \beta E_t \left[\frac{\pi_{t+1}^p - \beta E_t \pi_{t+2}^p}{\lambda_p} - \frac{\pi_t^p - \beta \pi_{t+1}^p}{\lambda_p} + \pi_{t+1}^p \right] - \lambda_w \left(\frac{\pi_t^p - \beta E_t \pi_{t+1}^p}{\lambda_p} - \left(\left(\sigma^{-1} + \varphi \right) c_t + \varphi g_t \right) \right)$$

Define $\tilde{\Delta}\pi_t^p = \pi_t^p - \beta E_t \pi_{t+1}^p$ and re-arranging gives:

$$[1 + \lambda_p + \lambda_w] \tilde{\Delta} \pi_t^p = \beta E_t \left(\tilde{\Delta} \pi_{t+1}^p - \tilde{\Delta} \pi_t^p \right) + \tilde{\Delta} \pi_{t-1}^p + \lambda_w \lambda_p (\left(\sigma^{-1} + \varphi \right) c_t + \varphi g_t)$$

$$c_t = E_t c_{t+1} - \sigma (i_t - E_t \pi_{t+1}^p) + \delta_t$$

$$i_t = \phi_\pi \pi_t^p + v_t$$

which are the equilibrium equations underlying the proof of Proposition 2 in the main text.

Appendix B Decreasing Returns to Scale and TFP Shocks

Under decreasing returns to scale and with TFP shocks (the benchmark model in the Gali and Woodford textbooks), the equations for production/ERC and marginal cost change to (we abstract from government spending shocks without loss of generality):

Production and ERC:
$$c_t = a_t + (1 - \alpha) n_t$$

Marginal cost: $mc_t = \omega_t - \frac{1}{1 - \alpha} a_t + \frac{\alpha}{1 - \alpha} c_t$

To prove the breakup of equivalence, it is sufficient to take the simplest special case where the two Phillips curves are static, $\beta = 0$ (the forward-looking term introduces an additional source of differences whenever the real wage is non-constant). The flexible-wage case $\lambda_w \to \infty$ now yields (under $\beta = 0$):

$$\pi_t^p = \lambda_p \left(\sigma^{-1} + \frac{\varphi + \alpha}{1 - \alpha} \right) c_t - \lambda_p \frac{1 + \varphi}{1 - \alpha} a_t$$

Whereas the flexible-price case $\lambda_p \to \infty$ implies

$$\pi_t^p = \lambda_w \left(\sigma^{-1} + \frac{\varphi + \alpha}{1 - \alpha} \right) c_t - \lambda_w \frac{1 + \varphi}{1 - \alpha} a_t + \frac{\alpha}{1 - \alpha} \Delta c_t - \frac{1}{1 - \alpha} \Delta a_t$$
$$= \lambda_w \left(\sigma^{-1} + \frac{\varphi + \alpha}{1 - \alpha} \right) c_t - \lambda_w \frac{1 + \varphi}{1 - \alpha} a_t - (\Delta c_t - \Delta n_t).$$

This proves that the wage Phillips curve always contains a term that captures the difference between consumption and hours growth, and this term is always non-zero insofar as either returns to scale are non-constant or there are disturbances to TFP.

It can be easily shown that a similar breakup of equivalence occurs in a model with endogenous investment in physical capital, where the Phillips curve under flexible prices features an extra term that is now equal to the difference between the growth rates of output and hours worked, $\Delta y_t - \Delta n_t$.